Experimental investigation on the dynamic modulus change of Natural Rubber under principal loading modes and different dynamic conditions.

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Abstract

The dynamic mechanical behavior of rubber material has been investigated for the principle loading modes; tensile, compressive and shearing. For each loading mode, specimens were subjected to sinusoidal cyclic loadings at two different mean strain states and two different frequency values. Each specimen has experienced 100K cyclic sinusoidal loading under the corresponding mean strain state and frequency. The amplitudes for cyclic loadings have been kept constant through the tests. Dynamic characterization tests were subjected to the specimens at an interval of 10K cycles during the cyclic loading tests. Dynamic characterization tests are also performed at the same mean strain states; however, the frequency value is swept between 1 and 101 Hz by a step size of 5 Hz. Also the dynamic load amplitude is continuously reduced as the frequency is increased in order to minimize the measurement errors at higher frequency values. The results are presented by means of comparative graphs. Curves are fit using appropriate functions to use the information gained experimentally in the finite element calculations. A methodology has been presented to estimate the dynamic mechanical behavior of vibration isolators which are produced from natural rubber and subjected to single mode loading.

1. Introduction

For decades, in many fields like, automotive, aviation, defense, electronics, ship construction, machine construction, concrete structure construction and road construction etc., elastomeric materials have been used by engineers because of their flexibility and energy absorption properties. The most consumed elastomeric material today is Natural Rubber (NR), also called Polyisoprene, in all over the world. The main reason of this fact is that NR has relatively greater mechanical properties compared to all other synthetic elastomeric materials. Although, the cost of NR gradually increases nowadays, it is still the most favorable material for rubber component manufacturers. One of the main uses of NR is to insulate the vibration which is encountered in almost every engineering problem. However, the prediction of dynamic mechanical properties of NR is difficult because of its dependency on both strain change and frequency. Moreover, its mechanical properties continuously change during the life span of the product, which is also an uncertainty for the engineer who wants to use rubber components in his/her system. In the past many engineers and scientists studied NR to express its static and dynamic mechanical properties by means mathematical expressions. Recently, Luo at al. [1] studied the frequency and strain amplitude dependence of dynamical mechanical properties and hysteresis loss of carbon black filled vulcanized natural rubber. Luo at al. took experimental measurements on the rubber strips loaded in tensile mode and compared the results with the outcomes of viscoelasticity. Tárrago et al. [2] experimentally investigated a commercially available rubber bushing and proposed a model to predict the axial and radial stiffness values for the bushing. Many constitutive models have been developed to express the dynamic mechanical behavior of rubber and rubber like materials in the past; hyperelasticity, viscoelasticity, Mullins effect [3] and Payne effect. Some authors focused on the effect of the amount of carbon black content in a rubber compound and studied the mechanical properties of rubber compounds of different chemical composition.

In the present study, three different rubber vulcanizate specimens are used to investigate the dynamic mechanical behavior of carbon-black reinforced natural rubber experimentally. Static and dynamic test results are collected to obtain the dynamic moduli and loss angle values for various mean strain states and frequency values. The re results are graphically presented and discussed. A parallel study was carried out using the frequency dependent dynamic moduli to make finite element calculations and compare the results with the experimentally obtained ones.

2. Experimental Arrangements and Calculations

Fig.1 shows the rubber cylinders, bonded to rigid bodies at both ends, used for compressive and tensile loadings. Fig. 1 also shows the rubber blocks bonded to steel plates at appropriate surfaces to investigate natural rubber in shear loadings. They all are produced in the same rubber injection press.

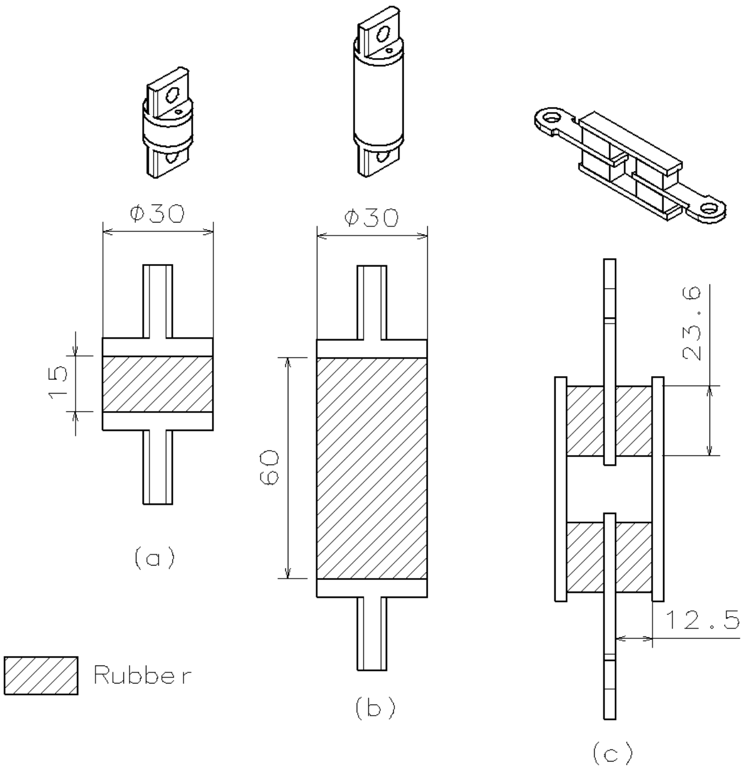


Fig. 1: Test Specimens; (a) compression, (b) tension, (c) shear, width=12.5 mm.

All tests are performed on the MTS 831.10 elastomer test system. The system has a load cell capacity of ±25kN and a piston stroke of 60mm. Piston movement can reach a maximum frequency value of 200 Hz at appropriate dynamic load/displacement amplitude.

Cyclic loading test conditions for each loading mode are given in the following table;

#	Loading mode	Mean strain	Dynamic strain amplitude	Frequency (Hz)	# of cycles
a	Comp.	0,2	0,067	5	100000
b	Comp.	0,4	0,067	15	100000
c	Shear	0,2	0,08	5	100000
d	Shear	0,4	0,08	15	100000
e	Tens.	0,2	0,017	5	100000
f	Tens.	0,4	0,017	15	100000

Table 1: Test conditions for cyclic loadings

In order to see the changes in the mechanical properties of rubber, static and dynamic characterization tests were carried out before the cyclic tests and after each 10000 cycles during the cyclic tests. Before each static test, a set of preconditioning cycles were performed to eliminate the Mullins effect (i.e. material softening due to strain change). Dynamic tests are also performed at the same mean strain state as it is set for the corresponding cyclic test. During the dynamic characterization tests, the frequency value is initially set to 1Hz. and swept up to 101 Hz by a constant sweep parameter of 5 Hz, the amplitude is initially set to 2mm (peak to peak) at 1Hz and reduced linearly to 0,1mm at 101Hz.

2.1. Compressive load tests

Under compressive loads, modulus of elasticity of rubber must be corrected before using in the stiffness formula. The following equation is used to find the static stiffness of a rubber block under a concentric compressive load;

$$K_c = \frac{E_c A}{t} \tag{1}$$

Because under compressive loads, geometric factors play an important role in the compressive modulus, E_c must be obtained using the following expression;

$$E_c = E_o\left(1 + 2\phi(SF)^2\right) \tag{2}$$

Above equation is valid for thick rubber blocks only [4]. *SF* can be defined as the ratio of load area to bulge area. For example, for a cylinder block whose radius is r and height is H, bulge area and load area can be given as $2\pi rH$ and πr^2, respectively. The incompressibility coefficient ϕ, and the original compressive modulus E_o are experimentally obtained. In order to obtain these coefficients an unbonded cylindrical rubber block -whose diameter and height are 30mm and 15mm, respectively- is subjected to a compressive load.

Similar expressions can be written to find the dynamic stiffness of a rubber block under a concentric compressive load;

$$K_c^* = \frac{D_c A}{t} \tag{3}$$

Where, D_c is the dynamic compressive modulus and K_c^* is the dynamic compressive stiffness under any dynamic loading condition. In this case, K_c can be measured and D_c expressed by the following expression;

$$D_c = D_o\left(1 + 2\phi(SF)^2\right) \quad (4)$$

Since, SF and ϕ does not change under both static and dynamic conditions, a ratio of dynamic to static modulus (RDSM) can be introduced as;

$$r_c = \frac{D_c}{E_c} = \frac{D_o}{E_o} = \frac{K_c^*}{K_c} \quad (5)$$

At the end, the ratio of dynamic compressive modulus to the static one is equal to the ratio of dynamic compressive stiffness to the static one. These stiffness values can easily be measured by using appropriate equipment.

2.2. Shearing load tests

When a rubber block is subjected to a shear load, its stiffness against this load can be expressed by the following term;

$$K_s = \frac{GA}{t} \quad (6)$$

Where,

G: static shear modulus,
A: area at which the load is being applied,
t: the height of the rubber block.

In a case that a dynamic load is applied, Eqn. 6 can be written as below;

$$K_s^* = \frac{G^* A}{t} \quad (7)$$

Where,

G^*: dynamic shear modulus,
A: area at which the load is being applied,
t: the height of the rubber block.

If the eqn. 7 is divided by Eqn. 6, the ratio of dynamic shear modulus to static shear modulus is obtained;

$$r_s = \frac{G^*}{G} = \frac{K_s^*}{K_s} \quad (8)$$

2.3. Tensile load tests

One can make a similar analysis, when a rubber block is subjected to a tensile load, its stiffness against this load can be expressed by the following term;

$$K_t = \frac{E_t A}{t} \tag{9}$$

Where,

E_t: static tensile modulus,
A: area at which the load is being applied,
t: the height of the rubber block.

In a case that a dynamic load is applied, Eqn. 9 can be written as below;

$$K_t^* = \frac{E_t^* A}{t} \tag{10}$$

Where,

E_t^*: dynamic shear modulus,
A: area at which the load is being applied,
t: the height of the rubber block.

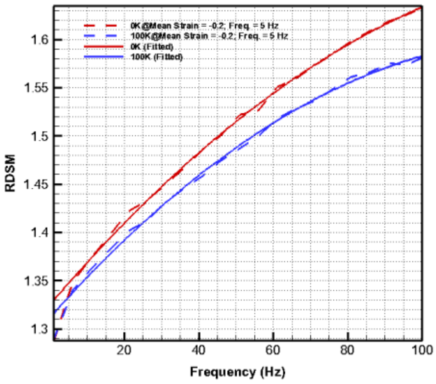
If the eqn. 10 is divided by Eqn. 9, the ratio of dynamic tensile modulus to static tensile modulus is obtained;

$$r_t = \frac{E_t^*}{E_t} = \frac{K_t^*}{K_t} \tag{11}$$

3. Results & Discussions

Two important dynamic mechanical characteristics of natural rubber may be at the interest of engineers and designers, who would like to use rubber components in systems; the change in the ratio of dynamic modulus to static modulus (RDSM) and the phase angle (PA) for each principle loading mode. The change in the PA is not to be discussed in the present study, because it is out the scope. The method of obtaining the RDSM for each loading mode was explained in the previous section.

Figures 2, 3 and 4 show the change in RDSM when a component is subjected to compressive, shear and tensile loads, respectively. As seen in fig. 2, when specimen “a” is undergone through a certain number of cyclic loadings under compressive loads, RDSM decreases either slightly or considerably depending on the mean strain state (see Figs. 2a, 2b, 2c and 2d). Less RDSM means reduction in the dynamic compressive modulus of NR. The area under RDSM curve can be used to express the change in RDSM after a cyclic loading, precisely. For instance, the change in RDSM of NR when it is subjected to cyclic loading under a mean strain state of -0,2 and at a frequency of 5 Hz is shown in fig. 2a. The corresponding change in the area under RDSM curve is given in Table 2 as -1,88%.



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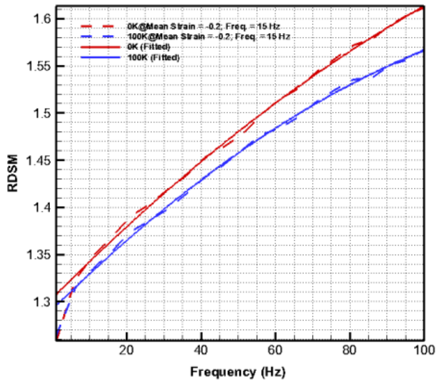

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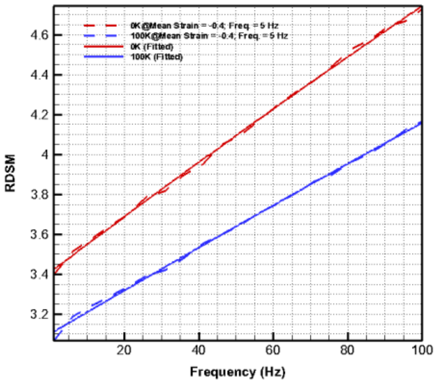

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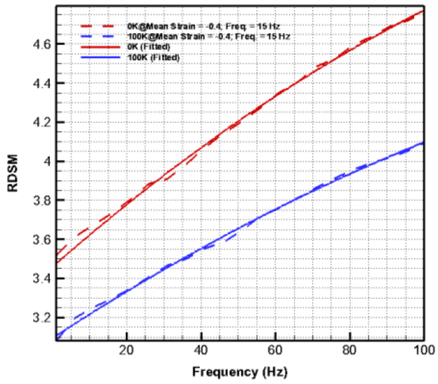

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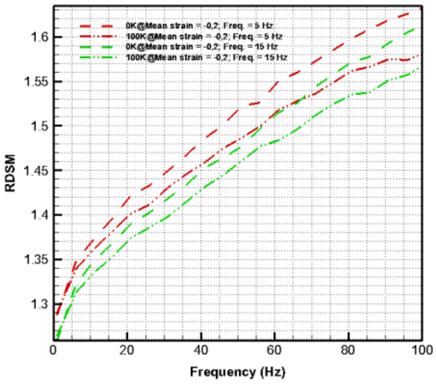

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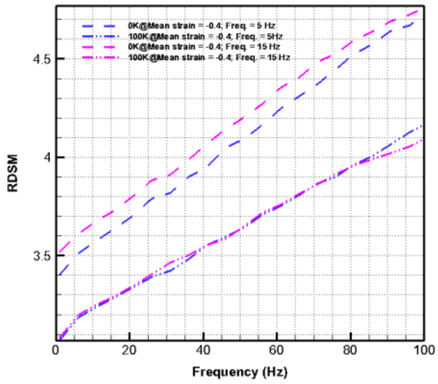

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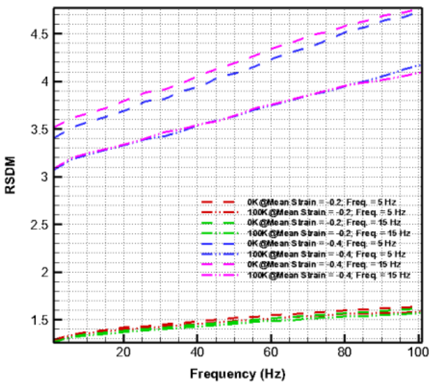

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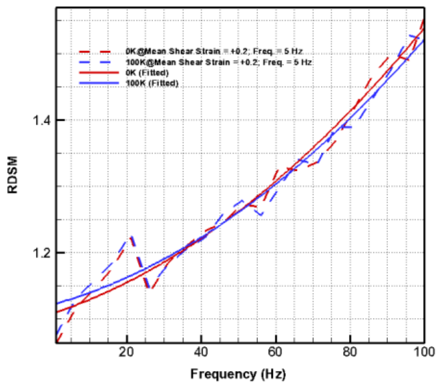
Fig. 2: Change in the ratio of dynamic modulus to static modulus (RDSM) under compressive loading



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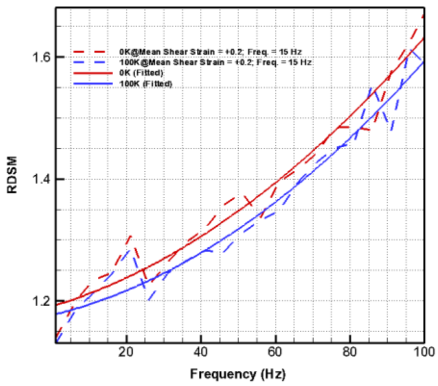

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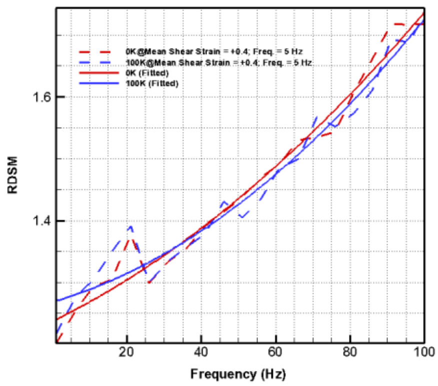

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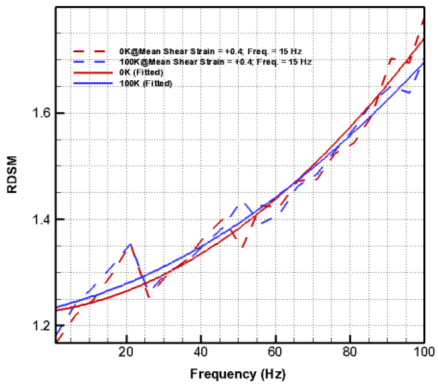

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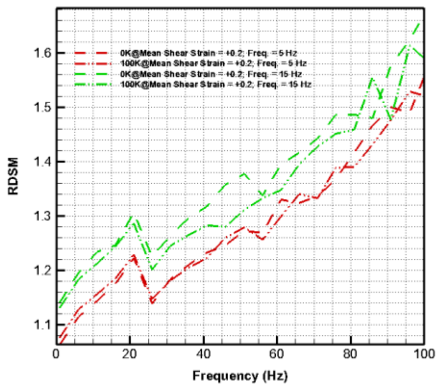

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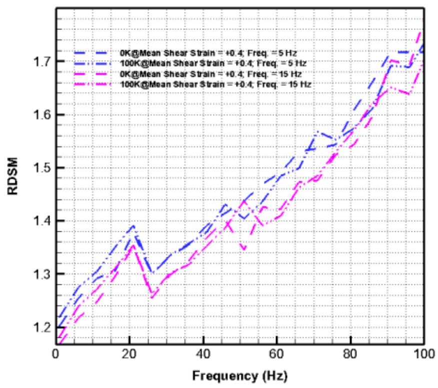

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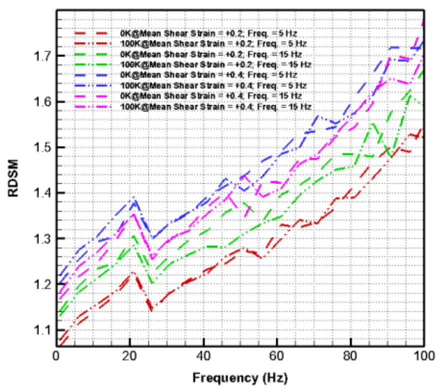

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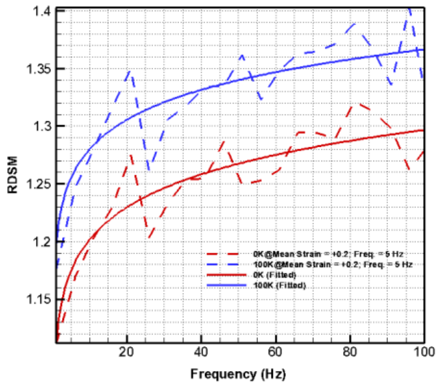
Fig. 3: Change in the ratio of dynamic modulus to static modulus (RDSM) under shear loading



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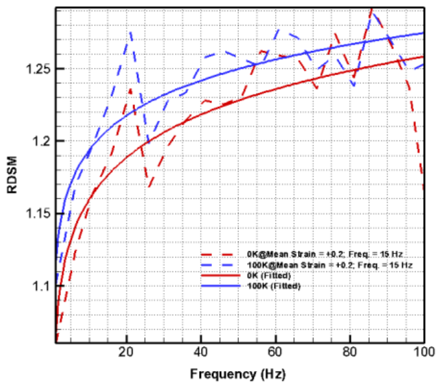

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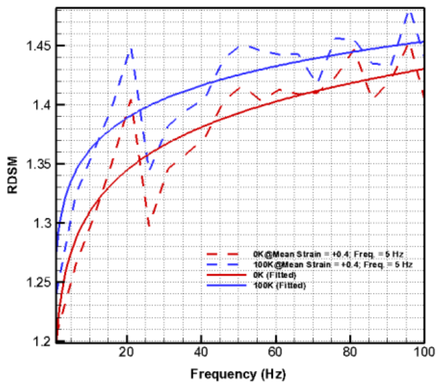

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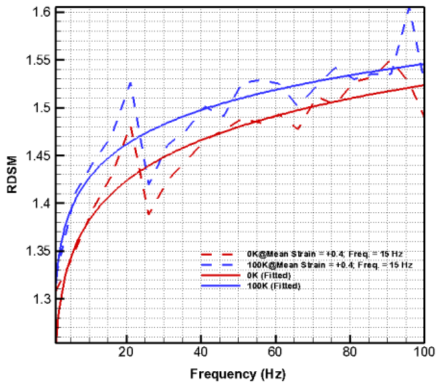

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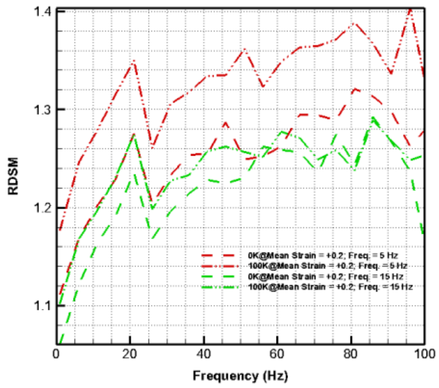

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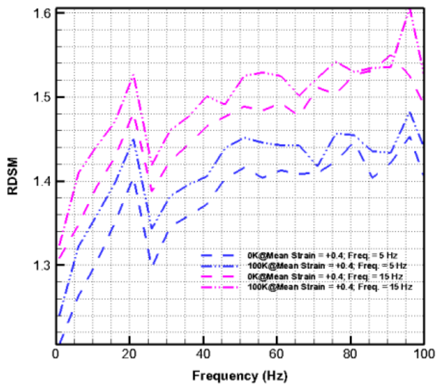

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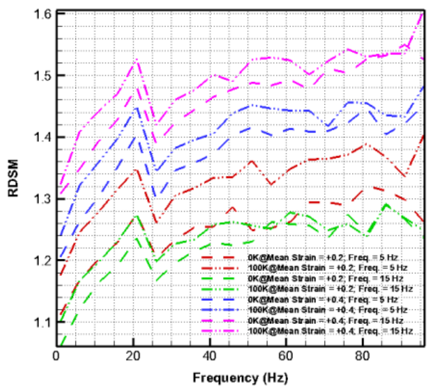

(g)

Fig. 4: Change in the ratio of dynamic modulus to static modulus (RDSM) under tensile loading

When the frequency value of cyclic loadings is increased to 15 Hz under the same mean strain state, the change in area reduces to -1,69%. Namely, the RDSM of NR compound at our interest reduces less when the frequency value is tripled from 5 Hz to 15Hz.

Cyclic test conditions	# of cycles	A	B	C	Area (unit^2)	Change in area (%)
	0K	1,3252	4,4979e-3	-1,4140e-5	1,5050e2	
ε_m=-0,2; *f*=5Hz	100K	1,3118	4,3247e-3	-1,6132e-5	1,4767e2	-1,88
	0K	1,3035	3,9639e-3	-8,6548e-6	1,4750e2	
ε_m=-0,2; *f*=15Hz	100K	1,2917	3,8796e-3	-1,1308e-5	1,4500e2	-1,69
	0K	3,4086	1,4071e-2	-7,2366e-6	4,1023e2	
ε_m=-0,4; *f*=5Hz	100K	3,0998	1,0979e-2	-3,9527e-6	3,6471e2	-11,09
	0K	3,5010	1,5018e-2	-2,1898e-5	4,1922e2	
ε_m=-0,4; *f*=15Hz	100K	3,1001	1,2161e-2	-2,1844e-5	3,6461e2	-13,03

Table 2: Curve fit parameters and change in the area enclosed by RDSM curve for compressive loads.

If the mean strain state is decreased to -0,4, the changes in the areas of RDSM curves become -11,09% and -13,03% for the frequency values of 5 Hz and 15 Hz, respectively. This means, lowering the compressive mean strain has more influence on the dynamic mechanical properties of NR than increasing the frequency value. When the compressive mean strain value is kept at -0,2, the change in the area of RDSM is slightly lower at 15 Hz than 5 Hz. In contrast, when the compressive mean strain value is reduced to -0,4, the change in the area of RDSM is lower at 5 Hz than 15 Hz. Therefore it can be said that the role of frequency under compressive cyclic loading becomes more effective if the mean strain value gets lower. Considering Fig. 2g, One can also make a judgment that mean strain value has the major influence on dynamic mechanical properties of NR compared to frequency under compressive loadings (i.e. the changes in the area of RDSM curves are -13,03% and -1,69% for the cyclic loadings ε_m=-0,2; *f*=15Hz and ε_m=-0,4; *f*=15Hz, respectively).

Cyclic test conditions	# of cycles	A	B	C	Area (unit^2)	Change in area (%)
	0K	1,1080	1,8390e-3	2,4753e-5	1,2875e2	
γ_m=+0,2; *f*=5Hz	100K	1,1212	1,5610e-3	2,4492e-5	1,2865e2	-0,08
	0K	1,1909	1,9279e-3	2,5903e-5	1,3734e2	
γ_m=+0,2; *f*=15Hz	100K	1,1765	1,4696e-3	2,7023e-5	1,3460e2	-2,00
	0K	1,2370	2,9417e-3	2,0581e-5	1,4594e2	
γ_m=+0,4; *f*=5Hz	100K	1,2683	1,7353e-3	2,8399e-5	1,4555e2	+0,14
	0K	1,2273	1,0540e-3	4,0894e-5	1,4220e2	
γ_m=+0,4; *f*=15Hz	100K	1,2327	1,7216e-3	2,9204e-5	1,4218e2	-0,01

Table 3: Curve fit parameters and change in the area enclosed by RDSM curve for shear loads.

Considering the graphs given in Fig. 3, it can be said that the mean strain dependence of dynamic mechanical properties of NR under shear loading is certainly less than compressive loadings. As it is reported in Table 3, the changes in the areas of RDSM curves are -0,08%, -2,00%, +0,14% and -0,01% for cyclic loading cases γ_m=+0,2; *f*=5Hz, γ_m=+0,2; *f*=15Hz, γ_m=+0,4; *f*=5Hz and γ_m=+0,4; *f*=15Hz, respectively. Here the maximum change obtained is -2,00% and the general tendency of NR under shear cyclic loadings is reduction in the dynamic mechanical properties similar to the results obtained for compressive cyclic loads. However, under shear loadings, NR at the interest has less mean strain dependency than compressive loadings, i.e. the change in the area of RDSM is -13,03% for the compressive cyclic loading of ε_m=-0,4; *f*=15Hz whereas, the change in the area of RDSM is -0,01% for the shear cyclic loading of γ_m=+0,4; *f*=15Hz. Comparing the RDSM curves shown in Figs. 3a, 3b, 3c and 3d, it can be concluded that NR is more durable under shear loadings than compressive loadings; the RDSM curves before and after the cyclic loadings are very close to each other hence the area change is relatively very small compared to the results of compressive cyclic loadings. Therefore, if a high durability is needed for a certain rubber component made of NR, it is suggested to use appropriate rubber geometries so that rubber is loaded in shear mode mostly.

Cyclic test conditions	# of cycles	A	B	Area (unit^2)	Change in area (%)
	0K	3,3088e-2	1,0749e-1	1,2569e2	
ε_m=+0,2; *f*=5Hz	100K	2,8431e-2	1,8137e-1	1,3307e2	+5,87
	0K	3,5538e-2	6,6184e-2	1,2203e2	
ε_m=+0,2; *f*=15Hz	100K	2,8486e-2	1,1145e-1	1,2417e2	+1,75
	0K	3,8455e-2	1,8082e-1	1,3799e2	
ε_m=+0,4; *f*=5Hz	100K	2,8355e-2	2,4342e-1	1,4145e2	+2,51
	0K	4,2367e-2	2,2606e-1	1,4668e2	
ε_m=+0,4; *f*=15Hz	100K	3,4886e-2	2,7491e-1	1,4981e2	+2,13

Table 4: Curve fit parameters and change in the area enclosed by RDSM curve for tensile loads.

Fig. 4 shows the change in RDSM when the test specimen “b” is subjected to tensile loadings. In practice, engineers are not advised to use any kinds of rubber material under tensile loadings. The results shown in Fig. 4 supports this advice because the curves have many undershoots and overshoots both before and after the cyclic tensile loadings. Therefore it can be said that NR at the interest has more unstable dynamic mechanical properties under tensile loadings compared to both shear and compressive loads. However in some applications like exhaust hangers, tensile loads are unavoidable. For such cases, either maximum or minimum limits should be defined in order to prevent failure or mall function. One can look at Figs. 4a, 4b, 4c and 4d and Table 4, and say if NR is subjected to a set of tensile cyclic loadings, its dynamic mechanical properties tend to increase. For instance, the areas under the RDSM curves are 1,2569x10^2 and 1,3307x10^2 before and after a cyclic tensile loading of ε_m=+0,2; *f*=5Hz, respectively. It means an increase of 5,87% in the area under RDSM curve.

In order to couple the experimental studies with finite element theory, appropriate curve fit functions can be used to determine the relation between dynamic modulus and frequency at a certain mean strain value. The fitted curves are shown in the first four graphs of Figs. 2, 3 and 4. The following second order polynomial curve fit functions are used for compressive loading and shear loading.

$$D_o = \left(A + Bf + Cf^2\right)E_o \tag{12}$$

$$G^* = \left(A + Bf + Cf^2\right)G \tag{13}$$

Where, *A*, *B* and *C* are the function coefficients and given in Tables 2 and 3.

However, the second order polynomial function is not suitable to express the dynamic tensile modulus in terms of frequency. Hence, power series function is used to determine the relation between dynamic tensile modulus and frequency at a certain mean strain value.

$$E_t^* = \left(e^{(A\log f + B)}\right)E_t \tag{14}$$

Where, *A* nad *B* are the function coefficients and given in Table 4.

In equations 12, 13 and 14, static modulii for compression, shear and tension are needed. A quasistatic tests are performed on specimens "a", "b" and "c" to determine the static modulii of the NR compound at interest. The results are given in the following table;

Static Modulii (MPa)	Engineering strain			
	0,1	0,2	0,3	0,4
E_o/v	2,37/0,69	2,83/1,14	3,29/1,65	3,75/2,17
G	0,70	0,66	0,62	0,59
E_t/v	2,11/0,51	1,50/0,13	1,19/-0,04	1,18/0,0

Table 5: Static modulii at several strain values (engineering).

The data given in above table is obtained from the engineering stress-strain curves, therefore the change in the cross-section is omitted. But it is interesting to compare the modulii results (see Table 6) obtained from true stress-strain curves with the engineering ones.

Static Modulii (MPa)	True strain			
	0,1	0,2	0,3	0,4
E_o/v	2,04/0,46	2,05/0,55	1,99/0,60	1,93/0,64
G	0,70	0,66	0,62	0,59
E_t/v	2,45/0,75	2,38/0,80	2,36/0,90	N/A

Table 6: Static modulii at several strain values (true).

Many studies carried out about the mechanical properties of rubber materials took into account the following relation;

$$E = 2(1+v)G \tag{15}$$

Where, *E* and *G* are the uni-axial modulus and shear modulus of rubber, respectively. *v* is called Poisson's Ratio. The above equation is only valid for materials whose mechanical properties are linear or can be assumed as linear. Rubber is classified as an incompressible material i.e. its Poisson's Ratio is 0,5. Hence, Eqn. 15 reduces to the following form;

$$E = 3G \tag{16}$$

Eqn. 16 is only valid for engineering stress-strain relations and means that modulus of elasticity of rubber is equal to three times of its shear modulus. Considering Table 5, this relation is only satisfied for tensile loads when the strain is equal to 0,1. Although the above relation is only valid for engineering stress-strain values, when it is applied to the modulus values obtained from true stress-strain data, the calculated values of Poisson's ratio get more consistent numbers under both compression and tension. The Poisson's ratio under compressive loads get more closer to the value of 0,5 if they are calculated by using true data. The modulii of rubber are very dependent on its strain, therefore as the strain changes, modulii either increase or decrease depending on the loading mode. For instance, in compression mode, the modulus increases as the absolute value of strain increases, however, the modulus decreases as the strain increases for tension mode. In case of shear mode, the modulus remains more stable than the other modes.

Engineers, who would like to use the NR material at the interest, can use outcomes of this study in finite element analyses as well. One can easily plug the expressions given in Eqns. 12, 13 and 14 to define the modulus of elasticity in terms of frequency. Therefore, dynamic mechanical properties for a component made of natural rubber can be estimated by a quite dependable deviation. The shear test was used to demonstrate the use data gained from this study in finite element theory. MSC Marc 2008 r1 is used to make the FEM calculations. Because the loads and geometry are symmetrical about the xy plane, the half of the specimen is modeled for simplicity in a three dimensional space as seen in Fig. 5. 1284 hexagonal elements with 8 nodes are used with a global edge length of 2 mm to construct the fem model.

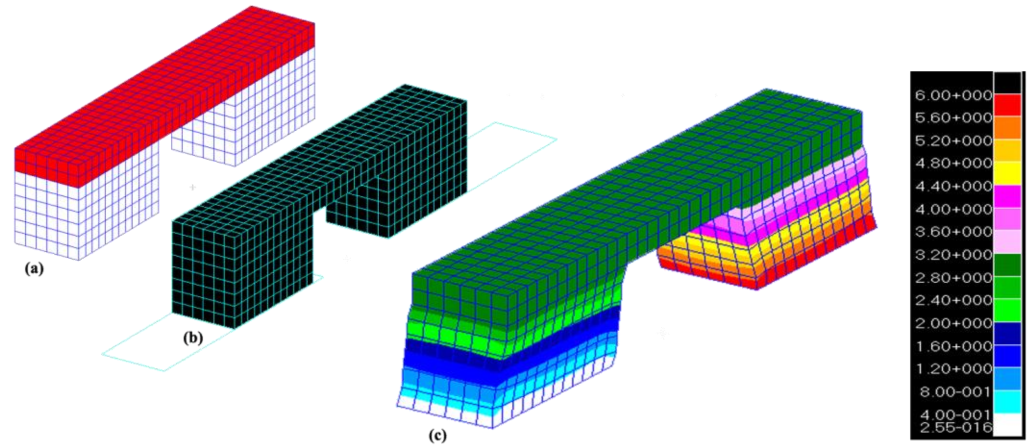

Fig. 5: Fem model of the shear test specimen.

The shear test specimens are produced of two different materials; natural rubber and steel. Therefore, steel part is also included in the fem model. Steel part is highlighted in Fig. 5a by the red color. Similarly, the natural rubber is highlighted by the white color in the same figure. Steel and rubber components are rigidly bonded to each other by using proper adhesion; hence in fem model they are modeled as bonded. Two rigid surfaces are also modeled to fix and move the model. One surface is kept stationary and the other is used to pre-load the rubber blocks so that the mean shear strain becomes 0.2 in the natural rubber blocks then moved dynamically at different frequencies at the same mean shear strain. By using such an approach, one can obtain both static and dynamic shear stiffness values by using the load displacement curves obtained from the finite element analysis. Thus, the RDSM's can be determined by using eqn. 8. The following table shows the fem results and their deviation compared to the measurements.

Property	f (Hz)				
	6	26	51	76	96
K_s^* (N/mm)	16,03	16,74	18,16	19,86	21,62
K_s (N/mm)	14,6	14,6	14,6	14,6	14,6
r_s (calculated)	1,10	1,14	1,24	1,36	1,48
r_s (measured)	1,12	1,17	1,27	1,39	1,51
Error (%)	-1,8	-2,6	-1,6	-2,2	-2,0

Table 7: Comparison of finite element and test results

As it can be seen from the table above, the results are very close to each other. However, the calculated RDSM values are observed to be slightly less than the measured ones. The maximum deviation between the calculated RDSM's and measured RDSM's became maximum -2,6% which is a quite dependable error considering how difficult it is to get reliable and dependable results from the finite element analyses of rubber components. So that if the relation between the dynamic modulus and frequency is known, one can easily implement it in the finite element analysis to estimate the value of dynamic stiffness of the rubber component with a considerably low error percentage.

Similar finite element calculations can be carried out for compressive and tensile loadings as well. But, considering the scope of this study it is enough to introduce the methodology by applying it to only shear loading mode. One can use the same methodology to predict or estimate the dynamic mechanical properties of natural or synthetic rubber materials.

4. Conclusions

Dynamic mechanical properties for a natural rubber compound at the interest of this study are determined and presented graphically under various dynamic loading conditions in compression, shear and tension. The changes in the ratio of dynamic to static modulii (RDSM) are introduced and discussed for all principal loading modes. Applicability of linear relations are discussed and a new methodology is presented to estimate or predict the dynamic stiffness value of a rubber component under a certain dynamic loading condition by using finite element analyses.